



Pyrford Church of England Primary Academy Trust

CALCULATION POLICY

Approval Date: 26 June 2015
Review Date: Summer 2018

Pyrford Church of England Primary School Academy Trust

Summary Statement of Intent

The Academy Trust was incorporated in November 2013.

Academy Members and Trust Directors acknowledge that Academy trusts are companies limited by guarantee and exempt charities. The Members and Trust Directors are subject to the duties and responsibilities of charity trustees and company directors, as well as any other conditions that are agreed with the Secretary of State. Members and Trust Directors recognise that these responsibilities are mutually reinforcing, to ensure the proper governance and conduct of the trust.

The key requirements are reflected in the Trust Articles of Association, the DFE funding agreement and the guidance of the Academies Financial Handbook. Members and Trust Directors accept the Charity Commissioners' guidance as to their statutory duties as company directors, which are set out in the Companies Act 2006. Members and the Trust Directors will incorporate the seven principles of public life in their decision making and demonstrate the proper stewardship of public funds for ensuring economy, efficiency and effectiveness in their use - the three key elements of value for money.

The Trust Directors are responsible for the strategic management of the school. This includes the Christian ethos of the school; its strategic vision and direction; annual budgets; senior staff appointments; and policy changes. Operational management is the responsibility of the headteacher and staff. It is carried out at a number of levels: i.e. Senior Leadership team, the Inclusion Manager and middle leaders.

The roles and responsibilities of the Trust Directors have been delegated to two committees: Efficiency, covering Finance, Personnel, Premises and Health and Safety; and Effectiveness, covering Standards and Curriculum, Safeguarding, Ethos and Admissions.

School policies are developed by senior members of staff, to reflect both the strategic direction agreed by Trust Directors and also the statutory and recommended policies appropriate to the school. These policies are approved and adopted by the Trust Directors, and implemented as procedures and systems by the Senior Leadership Team and other designated members of staff. These policies are available on the website and at the school.

Progression in Calculation Policy 2014

Children are introduced to the processes of calculation through practical, oral and mental activities with emphasis on practical in the first years at school. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.

It is important that, over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation.

As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and, importantly, have strategies to check its accuracy.

At whatever stage in their learning, and whatever method is being used, **children's strategies MUST still be underpinned by a SECURE and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.**

The overall aim is that when children leave Pynford Primary School they:

- have a solid bank of number facts and a good understanding of the four operations;
- have a secure understanding of how the four operations work and link together;
- know when they can perform a mental calculation using skills such as rounding, complements, doubles and halves
- have an efficient, reliable, written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- are confident with the rudiments of mathematics so that they can use and apply their knowledge quickly and accurately
- make connections between the different areas of numeracy so that they can reason mathematically and solve a range of simple and complex problems.

Number Acquisition / Counting

Early acquisition of number relies on learning number rhymes, counting in unison, noticing numbers around us.

During EYFS we focus on numbers from 1 to 20. We need to be secure with the number names and recognising the corresponding numerals. We need to be able to count on a given number of objects. We look for patterns.

During Year 1 we focus on numbers to 0-50. We need to be secure with the teen numbers and recognise the corresponding symbols. We start working with halves and quarters.

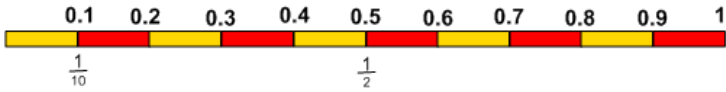
During Year 2 we focus on numbers 0-100. We work with $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$ and begin to look for equivalent fractions.

During Year 3 we start counting in tenths. We add fractions with the same denominator. We find equivalent fractions. We focus on unit fractions and those with small denominators.

From Year 4, we continue counting in decimals - to 3 decimal places - and we continue to develop understanding of fractions, improper fractions and mixed fractions.

Recite numbers in order	We count up and down in unison with others
1:1 pairing	<ul style="list-style-type: none"> - match cups to saucers - match hats to teddies - match buckets to spades etc.
Consolidate concept of cardinal numbers and counting	<ul style="list-style-type: none"> - drop coins into a cup counting 1, 2, 3, 4 - ask how many. - we need to be able to say how many without needing to re-count. <p>Play games: 'Guess how many'. Put out number of buttons (up to 10). Guess how many, then check. A key skill in number is to be able to estimate first.</p> <p>Dice games - recognise the dots on a spotty die first by counting them, then by instant recognition. This is a key skill.</p>
Recognising numerals	<p>Find all the number 1s hidden on a grid or in the sand.</p> <pre>@yh\$c&uy d5kakhbe' fnEjhi&n6 71?(82>n))p?'(_3e5</pre>
Conservation of number	<ul style="list-style-type: none"> - count out 5 things (dinosaurs, teddies, counters). Mess them up and move them around. We need to be able to say how many there are without needing to re-count.
Linking 1:1 pairing and cardinality	<ul style="list-style-type: none"> - put out 5 cups. Count how many cups. Can we put spoon into each cup and say how many spoons we have used? We should be able to say without counting. <p>Repeat with different numbers to 10 to check our understanding is</p>

	secure.																																																								
Ordinality - matching numerals to numbers of objects	<ul style="list-style-type: none"> - sets of towers and cards with numbers on. We match tower with number and then orders them. Ask, which number is 1 more than 4 etc. - we match Numicon pieces to numerals - we make / use Numicon number tracks/lines to show that the pieces increase by 1 each time. 																																																								
0 - 10 bead string 0-100 bead string	<p>Introduce the bead string. How many beads are there on the string? We show you different numbers as they are called out. "Show me 5" Remember: we return all the beads to the right of the string each time before we start a new number.</p> <p>We match beads on a string, to number shown on a card.</p>																																																								
Place Value	<p>We use Place Value mats and apparatus (place value counters, base 10) to represent numbers to help us understand our number system.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>100s</td> <td>10s</td> <td>1s</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> </tr> </table> <p>Year 1 In Year 1 we focus on numbers 11 - 20. Children find the teen numbers very difficult. We emphasise the 'teen' when saying the numbers (thirTEENO and we practise making the numbers using Numicon and base 10.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>100s</td> <td>10s</td> <td>1s</td> <td style="text-align: center;">●</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> <td></td> </tr> </table> <p>Year 2 We start to do our 'splitz' and partition numbers into 10s and 1s up to and beyond 100. We refer to the numbers as 100s, 10s and 1s (not H T U)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>1000s</td> <td>100s</td> <td>10s</td> <td>1s</td> <td style="text-align: center;">●</td> <td>$\frac{1}{10}$s</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>Year 3 We continue partitioning numbers and introduce 10ths.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>1000s</td> <td>100s</td> <td>10s</td> <td>1s</td> <td style="text-align: center;">●</td> <td>$\frac{1}{10}$s</td> <td>$\frac{1}{100}$s</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>Year 4 / 5 We introduce 100ths.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>100,000s</td> <td>10,000s</td> <td>1000s</td> <td>100s</td> <td>10s</td> <td>1s</td> <td>$\frac{1}{10}$s</td> <td>$\frac{1}{100}$s</td> </tr> <tr> <td style="height: 40px;"></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>Year 6 We count in steps of 1, 10, 100 or tenths and hundredths.</p>	100s	10s	1s				100s	10s	1s	●					1000s	100s	10s	1s	●	$\frac{1}{10}$ s							1000s	100s	10s	1s	●	$\frac{1}{10}$ s	$\frac{1}{100}$ s								100,000s	10,000s	1000s	100s	10s	1s	$\frac{1}{10}$ s	$\frac{1}{100}$ s								
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Bead string, bead bar and counting stick	<p>We use these to represent numbers smaller than 1 whole.</p> 
Counting	<p>We practise counting in equal steps:</p> <p>EYFS - count in 1s</p> <p>Year 1 - counting in 2s, 5s, 10s and towards end Year 1 halves</p> <p>Year 2 - counting in 2s, 5s and 10s and know our multiplication and associated division facts. Counting in halves and quarters.</p> <p>Year 3 - counting in 3s, 4s and 8s and know our multiplication and associated division facts. Counting in thirds, fifths, tenths.</p> <p>Year 4 - counting in different steps. Know all our multiplication and associated division facts for up to 12 x 12. Counting in tenths and hundredths.</p>

Progression in use of the number line

To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to be progression in their use of number tracks and number lines.

EYFS

Children start counting from 1, forwards and backwards, on a number track to 10. We then start from different places on the number track - not always 1, or 10.

1	2	3	4	5	6	7	8	9	10	11	12
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Never include 0 on a number track. We don't start counting from 0.

We extend to counting on a number track to 20.

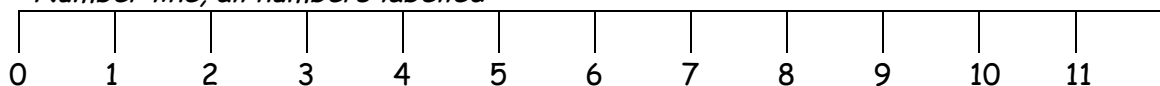
We use the 0-20 bead strings to count on and back - physically moving the beads as they count.



EYFS / Year 1

We move on to using a number line, where all the numbers are labelled. A number line to 100 will be displayed in the classroom in Year 1.

Number line, all numbers labelled



We move onto the **0-50 bead string** to locate numbers and to count forwards and back from numbers within 50. We place emphasis on learning the TEEN (Teen Queen) numbers.

We count in 10s using the bead string, emphasising the 'ty numbers (cup of tea numbers) - physically moving the beads as they count.

In Year 1, we use the number line (all numbers labelled) to show addition - practically only, encouraging starting with the biggest number first.

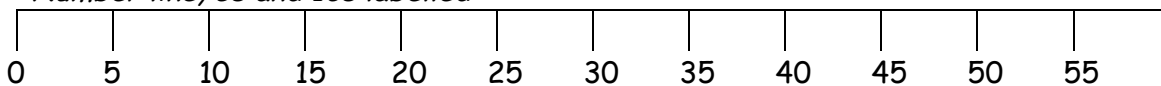
$$3 + 1 = 4$$



Year 1 / Year 2

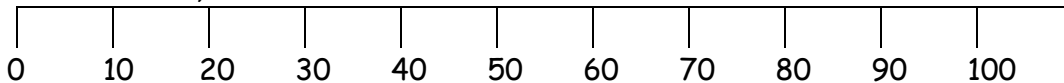
Towards the end of year 1, we use a number line where the 5s and 10s are labelled. We estimate where other numbers lie.

Number line, 5s and 10s labelled



We use a number line where 10s are labelled and estimate where other numbers lie.

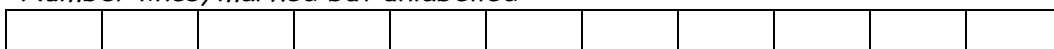
Number line, 10s labelled



Year 2

During Year 2, we move onto using a line where numbers are marked but unlabelled.

Number lines, marked but unlabelled



Year 2 / Year 3

The empty number line is introduced for jottings when performing calculations where it is difficult to keep all information in our heads.

Empty number line

Where we are using your complements to 10 to help us add - partitioning the number to get to the nearest 10, then adding on the remainder.
(i.e. $8 + 7 = 8 + 2 + 5$)

$$8 + 7 = 15$$



Progression in Teaching Addition

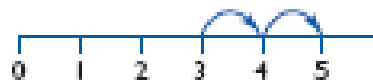
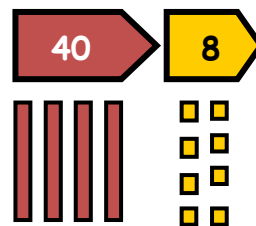
Mental Skills

- Recognise the size and position of numbers
- Count on in ones and tens
- Know number bonds to 10 and 20
- Know numbers bonds within 10
- Add multiples of 10 to any number
- Partition and recombine numbers
- Bridge through 10



Models, Images and apparatus

- Numicon
- Place value apparatus
- dienes, Place Value mat
- Arrow cards
- Number tracks
- Numbered number lines
- Marked but unnumbered number lines
- Empty number lines
- Hundred square
- Counting stick
- Bead string and beaded lines



Key Vocabulary

add

addition

Plus

and

count on

more

total

altogether

put together

increase

partition / splitz

carry

equal to

ADDITION

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

Children are entitled to be taught and to acquire secure mental methods of calculation and **one** efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers.

To add successfully, children need to be able to:

- recall all addition pairs within 10, (such as $\square + 3 = 10$);
- add mentally a series of one-digit numbers, (such as $5 + 8 + 4$) using facts they know such as near doubles, pairs to 10;
- add multiples of 10 (such as $60 + 70$) or of 100, (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- partition two, three and four-digit numbers into multiples of 1000, 100, 10 and 1 in different ways.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

We must explain that in mental calculation we always start with the biggest number so we are in the right area with our final answers, but when we are working in written algorithms we start with the 1s.

To add successfully, children need to be able to:

- recall all addition pairs to $9 + 9$ and complements in 10 (number bonds), such as $? + 3 = 10$.
- add mentally a series of 1-digit numbers, such as $5 + 8 + 4$
- add multiples of 10 (such as $60 + 70$) or of 100, (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value.
- partition 2-digit and 3-digit numbers (SPLIZ) into multiples of 100, 10 and 1 in different ways.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

EYFS

During EYFS we use a range of practical resources. Every opportunity is given to use those resources to combine numbers to find a total. We record using pictures.

The emphasis is on practically adding and EXPLAINING what we have done. We use lots of mathematical language.

Finding 1 more than a number	First this should be done practically on many occasions before recording is required.
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- Mummy had 4 apples. I gave her 1 more. How many does she have now?



She has 4 and 1 more, which is 5 altogether.

Introduce the + sign

We draw pictures to represent our work.

- 3 teddies were at the picnic. 1 more came along. How many teddies are there now?

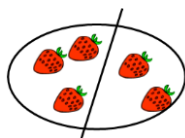


+

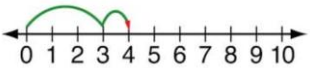
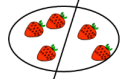


We use the Numicon number line to find the number that is 1 more than - recognising the pattern as it appears on the line.

Adding 2 single digit numbers



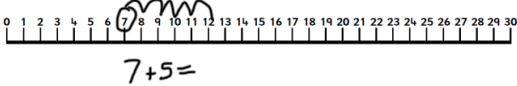
We represent our work with labels and pictures.

<p>Adding practically, using the symbols + = to record</p>	<p>Draw pictures with number labels $\odot\odot\odot + \odot\odot = \odot\odot\odot\odot\odot$ $3 + 2 = 5$ Make sets of 3 red cars and 2 blue cars or 3 red bricks and 2 green bricks. Ask how many bricks altogether? We use numeral cards to show the number sentence. Sometimes we copy the number sentence out.</p>
<p>Number track/line</p>	<p>We begin to use the number track to jump on 1 more. $3 + 1 = 4 =$ Find the first number, on the track, then jump on 1 more. Practise with games such as Snakes and Ladders. We try to remember to start with the biggest number. $3 + 1 = 4$  We progress to using number line to 20. Practically, put the biggest number in our heads, then count on...</p>
<p>Use number stories alongside early addition to reinforce.</p>	<p>One day Henny Penny laid 3 eggs and Turkey Lurkey laid 2 eggs. How many eggs were laid altogether? Get the children to write their own stories.</p>
<p>By this time children need to know:</p> <ul style="list-style-type: none"> number complements of numbers to 6 SECURELY and be able to partition these 	<p>$1+3 = 4, 2+2 = 4; 3+2 = 5, 1+4 = 5, 5+1=6, 4+2=6, 3+3 =6$ etc</p>  <p>Play dice games or pick a number card: roll the 0-6 spotty die. On a mat (see above) we count out that many dinosaurs or counters and then count out to find how many more make the target number.</p>

End EYFS / Year 1

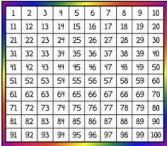

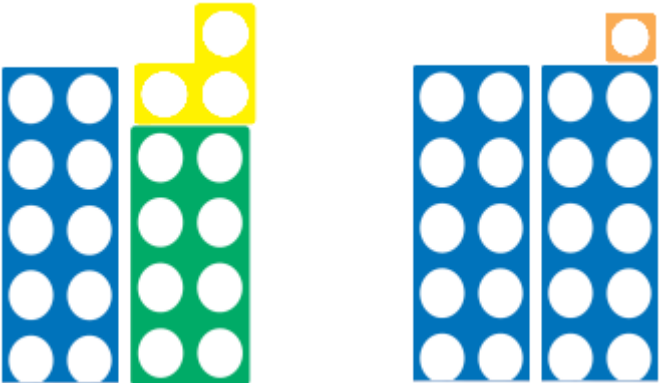
Children should continue to work practically throughout Year 1 and into Year 2. They should be encouraged to use apparatus whenever possible (counters, objects, bead strings, number lines, 100 grids).

They should continue to play games (Shut the Box; die games; Snakes and Ladders; complements to 10 pelmanism, dominoes to make 10 etc) to revise and secure number facts.

<p>Numicon - children should continue representing numbers using Numicon. This is good for subitising (recognising number as quantity). They should be able to select the appropriate piece of Numicon to make a number and make a number line using the Numicon. They should use Numicon to represent calculation and investigate equivalence using rocker balances with the = sign being put into the middle of the rocker stand to represent a number sentence.</p> <p>Children should begin to be able to say the value of each digit in any 2 digit number. They should begin to be able to recognise the purpose of 0 as a place holder, in practical situations.</p>	
<p>Introduce the numbered (landmarked line) (<i>rather than the number track</i>)</p>	<p>We use a number line, that we can record our calculation on.</p> <p>We begin to use a numbered number line.</p> <p>We start with a numbered number line before moving onto a beaded line.</p>  <p style="text-align: right;">So $7 + 5 =$</p> <p>This is done practically, but not recorded on the number line. It is a mental strategy. We only record the calculation itself.</p>
<p>0-50 bead string</p>	<p>We show you different numbers on the bead string as you call them, or have them written down.</p> <p>When working on numbers above 10, we lead onto moving 10 beads on, and then counting on the 1s (early place value).</p> <p>When making near 10 numbers, we look for strategies such as going to nearest 10 and counting back 1 to find, say 19, etc.</p> <p>We use a bead string to do addition of 2 single digit numbers, then move onto addition of single digit number to a 2 digit number (not bridging 10).</p> <p>We then move onto bridging 10.</p>
<p>Recording</p> <p>0-20 Beaded line and then 0 - 50 beaded line</p>	<p>We are still recording our work with models and images but now we should be moving onto using $+$ = to record number sentences for addition.</p> <p>We begin to locate specific numbers on a beaded line, where only the 10s are marked. They should be encouraged to count in 10s, then 1s to find numbers bigger than 10. They should be encouraged to develop strategies such as going to the nearest 10, then adjusting.</p> <p>We start to look for numbers on a 0-50 grid and try to spot patterns</p>

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

	<p>which will help us find numbers quickly. We then move onto the 100 grid.</p> <table border="1" data-bbox="828 315 986 394"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> </table> <p>The transition from physical beaded line to pictorial beaded line, can be difficult and requires much practise.</p> <p>Do not record addition on a beaded line, only use it for calculation - this is developing a mental strategy.</p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
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41	42	43	44	45	46	47	48	49	50																																										
Representing numbers	<p>The place value grid is introduced when looking at the value of digits in a number. We can use bundles of straws to represent 10s and 1s to make numbers. We can use sticks of 10 multilink cubes and 1s to represent numbers to 100.</p> <table border="1" data-bbox="1265 775 1465 902"> <tr> <td>100s</td> <td>10s</td> <td>1s</td> </tr> <tr> <td> </td> <td> </td> <td> </td> </tr> </table>	100s	10s	1s																																															
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<p>Number pairs within 10</p> <p>Complements to 10</p>	<p>In Year 1 we continuously rehearse the pairs of numbers that make 3, 4, 5, 6, 7, 8, 9 and 10.</p> <p>We begin to recognise that addition is commutative and can be done in any order so that if we know $1 + 9 = 10$ we also know that $9 + 1 = 10$. (We later relate these to the inverse, subtraction facts).</p> <p>We use the Numicon pieces and rocker balances and the equivalence scales to prove that the numbers can be partitioned but still have the same overall value (i.e. $3 + 2$ is the same as 5).</p> <p>We talk about = as the sign meaning "is the same as" "is equal to"</p> <p>We add 3 single digits together - looking for number pairs to help us perform this calculation mentally and efficiently.</p> <p>We initially do it by reordering the numbers and jotting it down.</p> <p>$3 + 4 + 7 =$</p> <p>$7 + 3 + 4 =$ because we know $7 + 3 = 10$ and then 4 more is 14</p> <p>This is continued and consolidated in Year 2</p>																																																		

<p>Counting in 10s</p>	<p>We introduce spider counting. Spiders can only go straight up and down on their web and so they love to count in 10s on a 100 grid. We use the spider counting to help us add 10, and then in Year 2 we can add multiples of 10 to a 2 digit number. This is a very useful pattern to recognise and a useful skill to use in mental calculation later.</p>  
<p>Showing addition</p>	<p>We add 2 numbers using Numicon, making lots of 10s and 1s.</p> <p>$13 + 8 =$</p> 
<p>Year 1 Fractions and Decimals</p>	<p>We recognise, find and name a half as one of two equal parts of an object, shape or quantity We recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</p> <p>We do lots of practical work with halves and quarters. We avoid using circles and use quadrilaterals as these are a better representation of fractions as equal parts of a whole and easier to work with.</p> <p>We use the words 'equal' 'equivalence' so that we become familiar with them.</p>

Problem solving

The bar model can be used to demonstrate addition when problem solving.

5 children get on the bus. At the next stop 8 more get on. This can be shown in the bar model to help the children recognise that they are needing to combine the two numbers (add them) to find the answer.

During problem solving, we should be encouraged to represent our work pictorially.
2 spiders and 1 dog. How many legs?



YEAR 2 - mental methods that lead to column addition

These generally involve partitioning numbers.

A key skill to cover at the outset of Year 2 is to partition numbers into 10s and 1s.

Base 10 should be introduced and referred to as 1s 10s 100s. Numbers should be made by placing the base 10 on the year 2 place value grid.

1000s	100s	10s	1s	•

This should be done alongside doing the SPLITZ - splitting the numbers into its 10s and 1s digits.

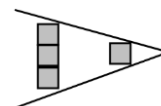
$$\begin{array}{c} 14 \\ / \quad \backslash \\ 10 \quad 4 \\ 10 + 4 \end{array}$$

Children should be able to say the value of each digit in any 2 digit number. They should begin to be able to recognise the purpose of 0 as a place holder.

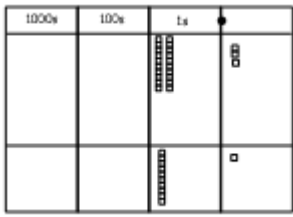



They should be able to count beyond 100 and compare and order numbers to 100 using < >

The crocodile should not be used to represent >

We represent it using practical resources (i.e. 2 hinged bars)



In Year 2 we practise rounding numbers to the nearest 10. This is an essential skill for mental calculation. Later we will use rounding to help us do efficient addition and subtraction but first we will do this on the number line, using Maths Frog to help us!

<p>We use the place value mat to show how the numbers combine, to find a total.</p>	<p>We place the first number above the line, and the 2nd below the line. We physically move the base 10 up to combine the two numbers. When necessary, we exchange 10 1s for a 10 and move it into the 10s column etc.</p>	
	<p>We begin to use a numbered number line to complete our calculation and we introduce math frog.</p>	 <p>Frog likes to do as few jumps as possible and so always jumps to the nearest 10 on the number line.</p>
<p>Empty Number Line</p> <p>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</p> <p>The empty number line helps to record the steps on the way to calculating the total.</p>	<p>Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. This is where we use Math Frog. He jumps to the nearest 10, and then adds the rest. This is for mental calculation.</p> <p>$8 + 7 = 15$</p>  <p>or</p> 	
<p>Using partitioning (Splitz)</p> <p>Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.</p>	<p>Record steps in addition using partitioning:</p> $47 + 76 =$ $7 + 6 = 13$ $40 + 70 = 110$ <p>Add the ones and then the tens to form partial sums and then add these partial sums.</p> <p>This leads onto formal column addition.</p>	
<p>Fractions and Decimals</p>	<p>We count up and down in halves. $\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 etc</p>	

In KS1, we NEVER present a calculation vertically. We always present them horizontally so that they are read correctly, and seen, as whole numbers.

YEAR 3

In Year 3 we continue to represent numbers on the 0-100 bead string or beaded line. We represent numbers using base 10 or place value counters on the place value grid.

1000s	100s	10s	1s	$\frac{1}{10}$

We continue to make links with apparatus and written methods CONTINUOUSLY, asking the children to show how they would calculate using the apparatus, as this shows they have conceptual understanding of the procedure.

Expanded method in columns

The expanded method leads children to the more compact method so that they can understand its structure and efficiency.

Write the numbers in columns.

- Move onto a layout showing the addition of tens to the tens and ones to the ones separately.

Adding the ones first: $58 + 47 =$

$$\begin{array}{r}
 50 + 8 \\
 40 + 7 \\
 \hline
 10 \\
 100 + 5 = 105
 \end{array}$$

We quickly move to the compact method of column addition.

Compact column method

Later, extend to adding 3digit numbers, two 3digit numbers and numbers with different numbers of digits.

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, this method is quick and reliable.

- In this method, recording is reduced further.
- **NOW WE LEAVE A LINE** after the last number in the calculation, and this is where we 'carry over' any digits.
- We use the words 'carry ten' or 'carry one hundred', NOT 'carry one'.

$$\begin{array}{r}
 258 \\
 + 87 \\
 \hline
 11 \\
 345
 \end{array}$$

Fractions and Decimals

We count up and down in quarters, halves.
 We count up and down in tenths and hundredths.
 We add fractions with the same denominator.
 We use our place value mats to represent decimals.

	<p>We round decimals with one decimal place to the nearest whole.</p>
<p>Year 4 Fractions and Decimals</p>	<p>We continue to count up and down in quarters, halves. We continue to count up and down in tenths and hundredths. We compare and order fractions whose denominators share the same multiple. We add fractions with the same denominator and denominators that are multiples of the same number. We round decimals with two decimal places to the nearest whole number and to one decimal place We read, write, order and compare numbers with up to three decimal places.</p>
<p>Year 5 Fractions and Decimals</p>	<p>We compare and order fractions whose denominators are all multiples of the same number We identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths We recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $2/5 + 4/5 = 6/5 = 1 \frac{1}{5}$] We add fractions with the same denominator and denominators that are multiples of the same number We read and write decimal numbers as fractions [for example, $0.71 = 71/100$] We recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents We round decimals with two decimal places to the nearest whole number and to one decimal place We read, write, order and compare numbers with up to three decimal places</p>
<p>Year 6 Fractions and Decimals</p>	<p>We use common factors to simplify fractions; use common multiples to express fractions in the same denomination We compare and order fractions, including fractions > 1 We add fractions with different denominators and mixed numbers, using the concept of equivalent fractions We identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places</p>

Progression in Teaching Subtraction

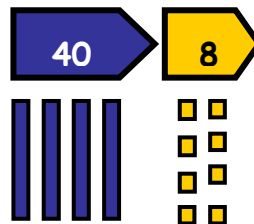
Mental Skills

- Recognise the size and position of numbers
- Count back in ones and tens
- Know number facts for all numbers to 20
- Subtract multiples of 10 from any number
- Partition and recombine numbers (only partition the number to be subtracted)
- Bridge through 10



Models, Images and Apparatus

- Numicon
- Place value apparatus
 - dienes and place value mats
- Arrow cards
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Hundred square
- Empty number lines
- Counting stick
- Bead strings



Key Vocabulary

subtract
 compare
 minus
 count back
 less
 fewer
 difference between
 distance between
 smaller than
 partition / split

count back take away
 fewer subtract
 minus less
 difference between

SUBTRACTION

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for subtraction of 2digit and 3digit whole numbers. These will be introduced to children AFTER they have understood the term subtraction by using objects and pictures to reinforce the idea. Children will always have had experience of using a numbered number line as well.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as $160-70$) using the related subtraction fact, $16-7$, and their knowledge of place value
- partition (SPLIT) 2-digit and 3-digit numbers into multiples of 100, 10 and 1 in different ways (e.g., partition 74 into $70 + 4$ or $60 + 14$)

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient method for subtraction.


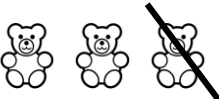


It is very important to use difference when teaching subtraction, because we are not always trying to take away something. For instance; I have £37 and I want to buy a radio costing £49. How much more do I need? We are finding the difference, not taking away, and if we don't introduce and use this term from the outset then children will find it confusing later.


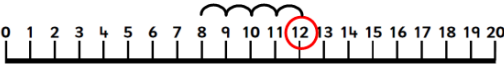
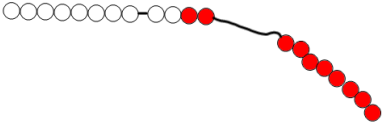
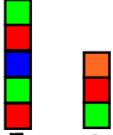
Do not use the term leave (i.e. 5 take away 3 leaves 2) because eventually children need to know that $5-3=2$ and $2=5-3$ but it won't make sense if the term 'leaves' is used.

Use 'compare' or 'how much bigger than' or 'how much smaller than' and 'is the same number as' and 'difference'.

EYFS

Early subtraction	Make set of 5 cars and remove 1. How many are there now? I had 9 apples but my rabbit ate 3 of them. How many do I have now? I had 5 sweets and I ate 2. How many sweets do I have for tomorrow? I had 9 apples. My friend Harry had 3 apples. How many more apples did I have? Use Numicon to represent this and compare the
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<p>Finding 1 less than a number</p> <p>Repeat practically and orally before recording pictorially.</p>	<p>numbers.</p> <p>First this should be done practically on many occasions before recording is required.</p> <ul style="list-style-type: none"> - Mummy had 4 apples. Tom ate 1 apple. How many does she have now? <div data-bbox="833 443 1453 515" style="border: 1px solid black; padding: 5px;">  <p>She has 4 and 1 less, is 3.</p> </div> <p>We draw pictures to represent their work.</p> <ul style="list-style-type: none"> - 3 teddies were at the picnic. 1 went home. How many teddies are there now? <div data-bbox="826 698 1045 795" style="text-align: center;">  </div> <p>We use the Numicon number line to find the number that is 1 less than - recognising the pattern as it appears on the line.</p> <p>We take a Numicon piece, and place a 1 piece on it and say how many are not covered.</p> <p>We use the 0-20 or 0-50 number line to count back.</p>
<p>Represent pictorially, alongside numerical labels. Introduce the - sign.</p>	<div data-bbox="837 1153 1292 1243" style="text-align: center;">  <p>$5 - 2 = 3$</p> </div>
<p>Year 1</p> <p>In Year 1, we start to learn our number pairs for all numbers to 10, including the inverse - subtraction. It is very important that children know their facts for, say, 4 :-</p> <p> $0 + 4 = 4$ $4 - 4 = 0$ $1 + 3 = 4$ $4 - 3 = 1$ $2 + 2 = 4$ $4 - 2 = 2$ $3 + 1 = 4$ $4 - 1 = 3$ $4 + 0 = 4$ $4 - 0 = 4$ </p> <p>We focus on showing links between addition and subtraction - associativity.</p>	
<p>Finding 1 less than a number</p> <p>Repeat practically and orally before recording pictorially.</p>	<p>We continue the work in EYFS but we move onto numbers up to 50.</p> <p>We use our bead strings and 0-50 number lines when finding 1 less.</p>
<p>Ask children to read aloud number</p>	<div data-bbox="845 1960 1037 2027" style="text-align: center;">  </div>

<p>sentences from their work. Ask them to make up number stories for it. Concentrate on 'take away' and 'less than'.</p> <p>We introduce difference practically when measuring length with cubes.</p>	<p>There were 5 leaves on a tree and 3 leaves fell off so there were 2 leaves hanging on the tree.</p> $3 + 2 = 5 \quad 5 = 3 + 2 \quad 5 - 3 = 2$ $2 + 3 = 5 \quad 5 = 2 + 3 \quad 5 - 2 = 3$ <p>We do lots of practical work with subtraction putting the counters into 2 sets to show that we are 'removing' (taking away) a subset to see what is left.</p> <p>7 cars and 3 drive off.</p>  <p>How many are left?</p>
<p>Once the children have practical experience of using a number line to subtract, they can record subtraction on a printed number line just to show the steps they have taken.</p>	<p>We begin to use a number line to subtract. First we put a circle around the first number, and then we jump back the number of jumps we need to subtract.</p> $12 - 4 =$  <p>We model this on a bead string to reinforce the concept of 'take away'.</p> 
<p>Finding 'the difference'.</p> <p>In Year 1, we do this by comparing sticks of multilink.</p>	 <p>5 3 The difference between 5 and 3 is 2.</p> <p>We practise this lots of times and then we move onto looking at the distance between 5 and 3 on a number line, to find the difference.</p>
<p>It is very important to use difference when teaching subtraction, because we are not always trying to take away something. For instance; I have £37 and I want to buy a radio costing £49. How much more do I need? We are finding the difference, not taking away, and if we don't introduce</p>	

and use this term from the outset then children will find it confusing later.

In KS1, we NEVER present a calculation vertically. We always present them horizontally so that they are read correctly, and seen, as whole numbers.

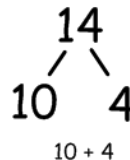
YEAR 2 - mental methods that lead to column subtraction

These generally involve partitioning numbers.

A key skill to cover at the outset of Year 2 is to partition numbers into 10s and 1s.

Base 10 should be introduced and referred to as 1s 10s 100s. Numbers should be made by placing the base 10 on the year 2 place value grid.

This should be done alongside doing the SPLITZ - splitting the numbers into its 10s and 1s digits.



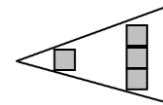
1000s	100s	10s	1s	•

Children should be able to say the value of each digit in any 2 digit number. They should begin to be able to recognise the purpose of 0 as a place holder.

They should be able to count forwards and back beyond 100 and compare and order numbers to 100 using $<$ $>$

The crocodile should not be used to represent $<$

We represent it using practical resources (i.e. 2 hinged bars)



In Year 2 we practise rounding numbers to the nearest 10. This is an essential skill for mental calculation. Later we will use rounding and complements to 10 to help us do efficient addition and subtraction but first we will do this on the number line, using Maths Frog to help us!

A key skill later in subtraction is knowing when to use counting on or or splitz to perform a calculation.

Example:

- $102 - 99 =$ it would be better to count on
- $52 - 21 =$ it would be better to take 20 and then take 1

We introduce Math Frog who likes jumping on a number line. But Math Frog doesn't like jumping too much because he gets tired so we always jump to the next 10 (using our complements to 10) and then do the rest.

$35 - 12 =$

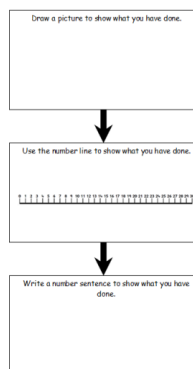


We move onto, jumping onto next 10 (using our complements to 10) and then jumping on the rest.

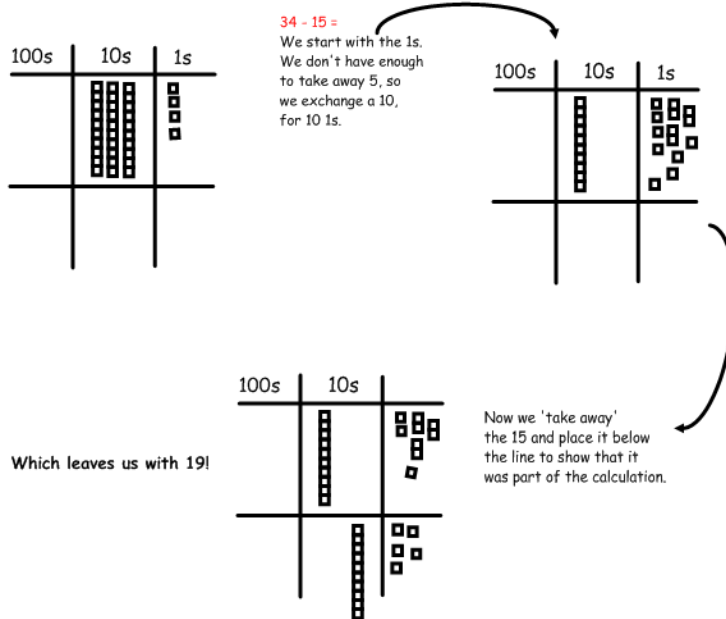
We don't record this on a number line, it is a practical exercise to help our mental calculations later.

With bigger numbers, we may later use an empty number line to record as jottings.

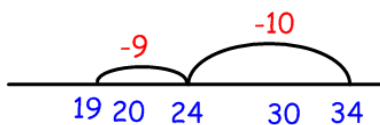
Encourage children to show the different ways that subtraction can be represented.



Subtraction using practical apparatus - place value grid, base 10.



This is modelled on the number line alongside to secure understanding of the processes involved.



YEAR 3

In Year 3 we continue to represent numbers on the 0-100 bead string or beaded line. We represent numbers using base 10 or place value counters on the place value grid.

1000s	100s	10s	1s	$\frac{1}{10}$

We continue to make links with apparatus and written methods *CONTINUOUSLY*, asking the children to show how they would calculate using the apparatus, as this shows they have conceptual understanding of the procedure.

Using partitioning (splitz) to subtract on a number line to support mental calculations.

With practice, children will need to record less information and decide whether to count back or forward.

It is useful to ask children whether counting up or back is the more efficient for calculations such as $57 - 12$, $86 - 77$ or $43 - 28$ for example.

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

$74 - 47 = 27$

We can use frog here!

or

Using partitioning (splitz) without a number line, to lead to expanded column subtraction.

This use of partitioning is a useful step towards the most commonly used compact column method.

Subtraction can be recorded using partitioning:

$$34 - 15 =$$

$$34 - 10 = 24$$

$$24 - 5 = 19$$

This requires children to subtract a single digit number or a multiple of 10 from a two digit number mentally.

It can be modelled using expanded column method

$$34 - 15 =$$

$$\begin{array}{r} 20 \quad 14 \\ 30 + 4 \\ - 10 + 5 \\ \hline 10 + 9 \end{array}$$

Using place value mats to subtract.

To continue to reinforce subtraction as taking away:

$34 - 15 =$
We start with the 1s. We don't have enough to take away 5, so we exchange a 10, for 10 1s.

Now we 'take away' the 15 and place it below the line to show that it was part of the calculation.

Which leaves us with 19!

When calculation with numbers close to a multiple of 100 or 1000 (if it is looking at you) then it is generally more efficient to use counting on (Math Frog), a mental method such as splitz or a number line.

100

i.e. $503 - 278 =$ count on from 278 to 300, then on from 300 to 503.

i.e. $341 - 202 = 341 - 200$, then subtract 2.

1000

Example

Year 4

We use the expanded method to subtract from 3 digit numbers and we model the compact method alongside

Example: $563 - 241$

$$\begin{array}{r} 500 + \cancel{60} + \cancel{3} \\ - \quad \quad 40 + 5 \\ \hline 500 + 10 + 8 = 518 \end{array} \quad 563 - 45 =$$

Start by subtracting the ones and then the tens.

Model with the place value mats and place value counters to aid understanding.

Ensure that children can explain the compact method, referring to the REAL value of the digits. They need to understand that they are repartitioning the $60 + 3$ as $50 + 13$.

<p>Year 3 Fractions and Decimals</p>	<p>We subtract fractions with the same denominator within one whole [for example, $7/7 + 1/7 = 6/7$] We compare and order unit fractions, and fractions with the same denominators</p>
<p>Year 4 Fractions and Decimals</p>	<p>We subtract fractions with the same denominator. We round decimals with one decimal place to the nearest whole number We compare numbers with the same number of decimal places up to two decimal places</p>
<p>Year 5 Fractions and Decimals</p>	<p>We subtract fractions with the same denominator and denominators that are multiples of the same number We recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents We round decimals with two decimal places to the nearest whole number and to one decimal place We read, write, order and compare numbers with up to three decimal places</p>
<p>Year 6 Fractions and Decimals</p>	<p>We compare and order fractions, including fractions > 1 We subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions</p>

Progression in Teaching Multiplication

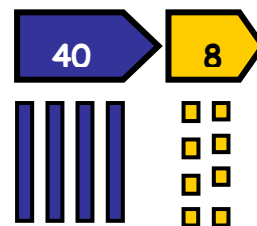
Mental Skills

- Recognise the size and position of numbers
- Count on in different steps 2s, 5s, 10s
- Double numbers to 10
- Recognise multiplication as repeated addition
- **Quick recall** of multiplication facts
- Use known facts to derive associated facts
- Multiplying by 10, 100, 1000 and **understanding the effect**
- Mutliplying by multiples of 10



Models, Images and Apparatus

- Numicon
- Place value apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines
- Multiplication squares
- Counting stick
- Models and images charts



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Key Vocabulary

Lots of

Groups of

Times

Multiply

Multiplication

Multiple

Product

Once, twice, three times

Array, row, column

Double

Repeated addition

multiplication **product**
once, twice, three times
double **groups of**
repeated addition **lots of**
array, row, column **multiply**
times **multiple**



MULTIPLICATION





The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

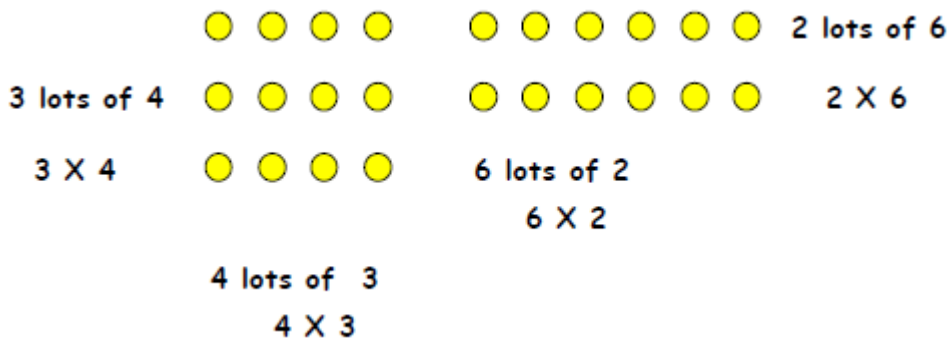
To multiply successfully, children need to be able to:

- recall all multiplication facts to 12×12
- partition numbers into multiples of one hundred, ten and one
- work out products such as 70×5 , 40×50 or 700×50 using the related fact 7×5 and their knowledge of place value
- add two or more single digit numbers mentally
- add multiples of 10 (such as $60 + 70$) using the related addition fact, $6 + 7$, and their knowledge of place value
- add combinations of whole numbers using the column method

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient method for multiplication.

<p>EYFS</p> <p>We will start to count forwards in 10s, flashing both our hands as we count. We will start to count forwards in 5s, flashing alternate hands as we count. We will look for patterns around us, as this will help us recognise patterns in the multiplication facts later.</p>	
<p>Recognising pattern</p>	<p>An important part of multiplication is recognising pattern.</p> <p>Continue the pattern with shapes, teddies, objects.</p>
<p>YEAR 1</p> <p>We count in 2s, 5s and 10s - forwards and backwards - in unison. We count in 2s, 5s and 10s to help find the total of a set. We stop occasionally and are asked, how many 2s? Show this with objects.</p> <p>We count 2p, 5p and 10p coins as they are dropped into a pot and we are asked, how many 10p, how much altogether?</p>	
<p>Recognising sets</p> <p>Children should arrange items as describe. Give them instructions on cards and they set them out. They could record as a picture and complete the blank</p> <p><input type="checkbox"/> sets of 2 4 sets of <input type="checkbox"/></p>	<p>3 sets of 2 socks </p> <p>2 sets of 3 leaves </p>

<p>Children need lots of practise at making and 'reading' sets.</p> <p>Question them: if you have 2 cars, how many wheels do you have? When counting the total say $2 + 2$ is 4 etc.</p>	<p>4 sets of 5 petals </p>
<p>Record the number sentence using + and the language: '2 and 2 is 4'</p>	<p> 2 sets of 2 is 4 $2 + 2 = 4$</p>
<p>Solve problems involving doubling.</p>	<p>Practically, if I have 5 fingers on one hand, how many do I have on two hands? I have 2 rows of 3 eggs in the box. How many eggs are there altogether?</p>
<p>YEAR 2</p> <p>We continue counting in 2s, 5s and 10s but now we also learn them as multiplication facts. $3 \times 2 = 6$</p> <p>We continue to represent multiplication with lots of apparatus: Numicon, base 10, cubes, arrays, things on plates.</p>	
<p>We represent multiplication as equal grouping, pictorially in a row</p> <p>$12 = 4 + 4 + 4$</p> <p>$12 = 2 + 2 + 2 + 2 + 2 + 2$</p>	<p> </p>
<p>Arrays</p> <p>Introducing the sign \times and calling it 'times'.</p> <p>Successful written methods depend on visualising multiplication as a rectangular array. It also helps children to understand why 3×4 is the same as 4×3. It is the first step in multiplication, as it is a practical application of the grid method.</p>	



We can use a folding arrays mat and talk and make the different arrays given.

i.e. Show me an array that is 5 along and 3 down. How many are there altogether? What number sentence could we write to record this? 5 lots of 3 3 lots of 5 5×3 3×5

Looking for pattern:

This model illustrates how multiplication relates to repeated addition.

Pattern work on a 100 square helps children to recognise multiples and rules of divisibility.

Using a bead bar, or counting stick:

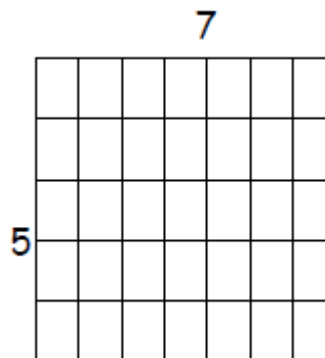


The rectangular array gives a good visual model for multiplication.

The area can be found by repeated addition (in this case $7 + 7 + 7 + 7 + 7$)

Children should then commit 7×5 to memory and know that it is the same as 5×7

$7 \times 5 = 35$



Multiplying numbers by 10.

We use base 10, or place value counters, and our place value mat to show **how the numbers move** along as they become 10 times bigger. We have a decimal point on our place value mat, so that we get used to seeing it there as a fixed point.

1000s	100s	10s	1s	.

YEAR 3

We continue to learn our multiplication facts for the 2x 5x 10x tables.

In Year 3, we need to know our 3x 4x and 8x facts by heart.

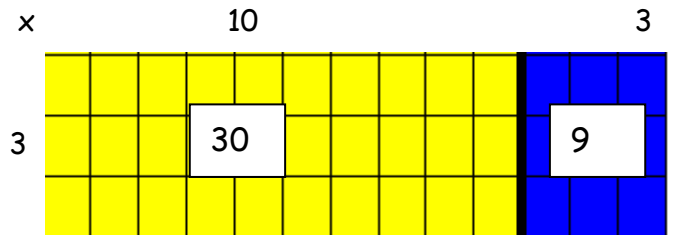
We continue to use the model of the array to show multiplication facts.

Mental multiplication using arrays and partitioning to multiply a 2digit number by a friendly 1digit number (x1, x2, x3, x5)

(Please note that the squares are used to ensure that the children have a secure mental image of why the distributive law works).

$$13 \times 3$$

We use splitz to partition the numbers to help us perform our calculation.



We can use the base 10 pieces to make a practical model showing an array.

Using the 'blank grid' to show arrays.

$$21 \times 13 =$$

It is important that children can multiply multiples of 10 to use this compact grid method.

$$21 \times 13$$

Step 1: use dienes to perform algorithm

Step 2: put in the numbers.

x	20	1
10		
3		

Multiplying numbers by 10.

We use base 10, or place value counters, and our place value mat to show **how the numbers move** along as they become 10 times bigger. We have a decimal point on our place value mat, so that we get used to seeing it there as a fixed point.

1000s	100s	10s	1s	.	$\frac{1}{10}$

YEAR 4

We continue to learn and use our multiplication facts for 2x, 5x, 10x, 3x, 4x, 8x tables.

We now learn the remainder of facts up to 12 x 12 so that we have instant recall.

We check we know the associated division facts.

We continue to use the grid method of multiplication and extend to multiplying a 3 digit by a 1 digit number.

We use the expanded method of multiplication as a way of working towards the short column method of multiplication.

Expanded method of multiplication, partitioning the numbers as in the grid method.

We multiply 2 and 3 digit numbers by a single digit number using this method.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 56 \quad (8 \times 7) \\ 210 \quad (30 \times 7) \\ \hline 266 \quad (\textit{remember to leave a line in case any carrying is required!}) \end{array}$$

YEAR 5

We continue to rehearse and use our multiplication facts in a number of ways - checking we know the associated division facts.

We move onto the **compact column method - short multiplication.**

We multiply 2 and 3 digit numbers by a single digit number.

In all instances we use our estimation/approximation skills to check for reasonableness of answers.

38×7 is approximately $40 \times 7 = 280$

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \quad (\textit{remember to leave a line in case any carrying is required!}) \end{array}$$

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a 2digit or 3digit number mentally before they reach this stage.

Long multiplication for multiplying 2digit by 2digit numbers includes the working to emphasise the link to the grid method

We continue to use the grid method until we are secure enough to move onto the long multiplication method.

56×27 is approximately $60 \times 30 = 1800$

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (6 \times 7) \\ 350 \quad (50 \times 7) \\ 120 \quad (6 \times 20) \\ 1000 \quad (50 \times 20) \\ \hline 1512 \end{array}$$

<p>Year 6 If we are secure with long multiplication we can move onto compact method for TU x TU and HTU x TU.</p>	$ \begin{array}{r} 23 \\ \times 12 \\ \hline 46 \\ 230 \\ \hline 276 \end{array} $																				
<p>Fractions and Decimals When multiplying decimals, the grid method works just as well.</p>	<p>38.5×24 is approximately $40 \times 25 = 1000$</p> <table border="1" data-bbox="667 651 1174 898"> <tr> <td>X</td> <td>30</td> <td>8</td> <td>0.5</td> <td></td> </tr> <tr> <td>20</td> <td>600</td> <td>160</td> <td>10</td> <td>770</td> </tr> <tr> <td>4</td> <td>120</td> <td>32</td> <td>2</td> <td>154</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>924</td> </tr> </table>	X	30	8	0.5		20	600	160	10	770	4	120	32	2	154					924
X	30	8	0.5																		
20	600	160	10	770																	
4	120	32	2	154																	
				924																	

Progression in Teaching Division

Mental Skills

- Recognise the size and position of numbers
- Count back in different steps 2s, 5s, 10s
- Halve numbers to 10
- Recognise division as repeated subtraction
- Quick recall of division facts
- Use known facts to derive associated facts
- Divide by 10, 100, 1000 and understand the effect
- Divide by multiples of 10



Models, Images and Apparatus

- Counting apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines
- Multiplication squares
- Models and Images charts

Key Vocabulary

- | | |
|-----------|----------------|
| Lots of | Groups of |
| Share | Shared between |
| Group | |
| Divide | Divide into |
| Division | Divided by |
| Remainder | |
| Factor | |
| Quotient | |
| Divisible | |



group groups of

lots of divide

divided by quotient

division factor

remainder divisible

half halve share

DIVISION

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence.

At the heart of success of division is clearly the mastery of times tables. The more fluent a child is with their tables, the easier they will find division. For this reason, when learning our multiplication facts we learn the associated division facts alongside.

To divide successfully, in their heads, children need to be able to:

- understand and use the vocabulary of division
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
- recall multiplication and division facts to 12×12
- recognise multiples of one digit numbers and divide multiples of 10 or 100 by a single digit number using their knowledge of division facts and place value
- know how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
- understand and use multiplication and division as inverse operations.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successfully, children also need to be able to:

- understand division as repeated subtraction (grouping)
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92
- know subtraction facts to 20 and to use knowledge to subtract multiples of 10 e.g., $120-80$, $320-90$

EYFS / Year 1

We provide many practical experiences for division as sharing and grouping, encouraging discussion of the concepts and ideas that are taking place.

We practising grouping things into 2s. We talk about odd and even numbers.

Sharing:

We know how many groups there are, but not how many are in each group. The answer is the number in each group.

Children share objects and use appropriate language to explain what they are doing.

i.e. There are 6 lollies and I'm sharing them with Tom and Harry so we are getting 2 each.



There are 7 lollies and I'm sharing them with Harry, so we are getting 2 each and there's 1 left over.



Grouping:

We know how many are in each group, but now how many groups there will be. The answer is the number of groups.

In KS1 we need to do lots of grouping to help children to understand relationship between times tables and division.

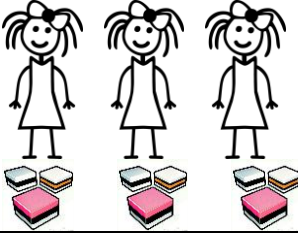

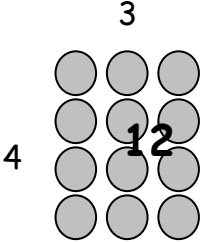
Children again have a range of objects but this time it is approached differently.

i.e. There are 6 lollies and 3 children, so how many lollies can they each have? How many lots of 3 are there in 6?



Grouping is linked more directly to multiplication and associated division facts.

Both mathematical terms are used practically so that children begin to see the links.

<p>Equal sharing</p> <p>There should be lots of practical experiences of 'sharing' - share these sweets between the girls etc.</p> <p>Move onto representing with pictures</p>	<p>Practically sharing cups between the teddies; toys between the children; people between the cars etc.</p> 
<p>Equal grouping</p> <p>I have 12 sweets and I want to give each person 3. How many people will get 3 sweets?</p> <p>Lots of practical experience is needed. Then represent with pictures.</p>	
<p>Solve problems involving halving and sharing</p>	<p>I have 4 sweets and I share them - half for you, half for me. How many do we each get?</p>
<p>When sharing and grouping practically, you can include numbers with remainders and use this as an opportunity for discussion.</p>	
<p>Year 1 Fractions - a half and a quarter</p> <p>It is important to introduce the terms half and a quarter when sharing and children can record their work but do not introduce the symbols $\frac{1}{2}$ $\frac{1}{4}$ too early. It should be a half of 8 is 4 etc.</p>	
<p>Year 2</p> <p>We learn the division facts associated with the 2x 5x and 10x tables.</p> <p>We continue to represent division with lots of apparatus: Numicon, base 10, cubes, arrays, things on plates.</p> <p>We introduce the \div sign and we focus on grouping, where we find how many groups there are i.e. $56 \div 8 =$ we find how many groups of 8 there are in 56, therefore using our multiplication facts.</p>	
<p>Using arrays for division</p> <p>$12 \div 3 =$</p> <p>It is important to make the links between multiplication and division so that the children can use their table facts to help them solve problems.</p> <p>Encourage the children to draw arrays to solve problems.</p>	

YEAR 3

We learn the division facts associated with the 2x 5x and 10x tables and now the 3x, 4x and 6x tables.

We continue to represent division with lots of apparatus: Numicon, base 10, cubes, arrays, things on plates but we now also represent division with the place value mat.

$363 \div 3 =$ How many groups of 3 hundreds can I make etc?

1000s	100s	10s	1s	$\frac{1}{10}$ s
	3	6	3	

We use arrays to represent division and this leads onto the grid method.



$$56 \div 7 =$$

How many 7s can I see? (grouping)

If I put these into 7 groups, how many in each group? (sharing)



We introduce the chunking method, where we use repeated subtraction to help us answer division questions.

Dividing numbers by 10 or 100.

This is a key fact and must be taught using place value mats and counters or base 10 to give the children a visual image.

In Year 3, focus on dividing by 10.

We use base 10, or place value counters, and our place value mat to show **how the numbers move** along as they become 10 times smaller. We have a decimal point on our place value mat, so that we get used to seeing it there as a fixed point.

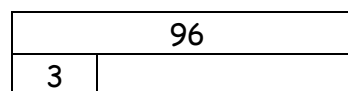
We work with whole numbers initially.

1000s	100s	10s	1s	$\frac{1}{10}$

The grouping method for $TU \div U$

$$96 \div 3 =$$

First, use the bar model to show the calculation in context:



Effectively we are asking, how many groups of 3 can we make with 96.

We can use Cuisenaire to calculate the answer.

Find 96 on the number track.

Use the 3 rod and keep placing them on the number track until you get to 96. It is recorded horizontally as $96 \div 3 = 21$

For questions there are remainders, the visual

representation makes this clear.

Year 4

We continue to learn and use our multiplication facts and associated division facts for 2x, 5x, 10x, 3x, 4x, 8x tables.

We now learn the remainder of facts, with associated division facts, up to 12 x 12 so that we have instant recall.

We continue to use the place value mats and arrays to help us represent division with images.

We find the effect of dividing a 1 or 2 digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

By the end of the year, children should be able to divide a 3 digit number by a single digit number with no remainders.

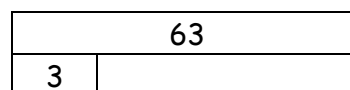
We continue modelling division using place value mats, regrouping 100s for 10s, or 10s for 1s.

We continue practising multiplying and dividing by 10 or 100 and discuss the effects on the original number.

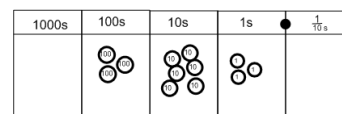
The grouping method for $TU \div U$ as in Year 3 this is done practically, but now recording as compact method is introduced alongside.

$$96 \div 3 =$$

First, use the bar model to show the calculation in *context*:



Now we use the place value mat and dienes/place value counters to find how many groups of 3 can I make out of 74?



The calculation is **recorded** using the bus stop (compact) method of division.

$$\begin{array}{r} 21 \\ 3 \overline{) 63} \end{array}$$

Firstly, done with no carrying involved.

When secure, introduce numbers which require exchanging/regrouping. Again use the place value mat and dienes/place value counters. Again it is **recorded** -not calculated using the bus stop method.

	$\begin{array}{r} 21 \\ 3 \overline{) 42} \end{array}$												
<p>The Chunking Method for HTU ÷ U</p> <p>The key to the efficiency of chunking lies in the estimate that is made before the chunking starts.</p> <p>Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.</p> <p>Estimating has two purposes when doing a division: to help to choose a starting point for division to check the answer after the calculation.</p>	<p>To find $196 \div 6$,</p> <p>we start by writing a fact box using our multiplication facts and then we use those facts to repeatedly subtract, until we have none left:</p> <p>Initially children will subtract chunks about which they are totally confident. Here a series of chunks (6x10) are subtracted to reach 16 then 6x2 until no more whole sixes are left, leaving a remainder of 4</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $\begin{array}{r} 196 \\ - 180 \quad (30 \times 6) \\ \hline 16 \\ \quad 12 \quad (2 \times 6) \\ \hline 4 \quad 32 \end{array}$ <p>Answer is 32 r 4</p> </div> <div style="text-align: center;"> <p>Fact Box</p> <table style="border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">1</td><td style="padding: 2px 10px;">6</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">2</td><td style="padding: 2px 10px;">12</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">3</td><td style="padding: 2px 10px;">18</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">4</td><td style="padding: 2px 10px;">24</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">5</td><td style="padding: 2px 10px;">30</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 10px;">6</td><td style="padding: 2px 10px;">36</td></tr> </table> </div> </div>	1	6	2	12	3	18	4	24	5	30	6	36
1	6												
2	12												
3	18												
4	24												
5	30												
6	36												

YEAR 5

We continue to rehearse and use our multiplication and division facts in a number of ways, ensuring we have quick recall.

We continue with the short division method for division of 2, 3 and 4 digit numbers by a single digit.

We use multiplication and division as inverses to support the introduction of ratio in Year 6, e.g. in scale drawings or in converting between metric units. e.g. 1m = 100 cm so 7m = 700 cm.

We continue to use place value mats and counters to model division and particularly when talking about remainders.

The emphasis in Year 5 is with manipulating the remainder. Using division within problem solving makes the remainder 'real'.

Bus shelter method / Short Division

Children should first be introduced to this method by working through calculations with no remainders.

Children should then solve calculations with remainders.

Now we are secure with the compact method, we can go straight into calculating using it and using practical apparatus only to check our answers.

$$693 \div 3 =$$

$$\begin{array}{r} 231 \\ 3 \overline{) 693} \end{array}$$

For example: $937 \div 3 =$

$$\begin{array}{r} 312 \text{ r } 1 \\ 3 \overline{) 937} \end{array}$$

Long Division – for more complicated calculations

This is used when calculating HTU \div TU until we are ready to use short division for HTU \div TU.

Autumn term: remainders just as remainders

Spring term: remainders as a decimal

Summer term: remainders as a fraction

When teaching long division, we show the short division by the side so that children can see that short division is in fact much simpler, quicker, easier and more efficient.

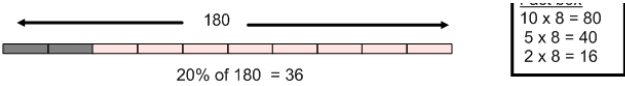

$$560 \div 24 =$$

We need a fact box. In the fact box we write down the multiplication facts for the divisor (i.e.

We start by subtracting 480 from 560.

$$\begin{array}{r} 23 \text{ r } 8 \\ 24 \overline{) 560} \\ \underline{48} \quad (2 \times 24) \\ 80 \\ \underline{72} \quad (3 \times 24) \\ 8 \end{array}$$

Fact Box	
1	24
2	48
3	72
4	96
5	120
6	144

<p>Using the place value mat, makes the decimal notation much easier.</p>	
<p>Percentages In Year 5 we need to know % equivalents of $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.</p>	<p>We use the bar model to illustrate what we are calculating.</p>  <p>20% of 180 = 36 and we can use a Fact Box to note down what we know will help us.</p> <p>A new cook started in the bakery. On the first day he burned 625 cookies. If that was 25% of the cookies made during the day, how many were made in total?</p> 
<p>Year 6 We now divided numbers up to 4 digits by a 2 digit number using long division.</p> <p>We interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context.</p>	
<p>Fractions and decimals.</p>	<p>It is important to use visual images to support division with fractions.</p> <p>Children need to be secure with dividing by 10 and 100.</p> <p>We use paper strips or the number line to help model fractions - this helps us to visualise what we are doing. i.e. $1 \div \frac{1}{2} =$ use paper strip to show how many halves make 1 whole, or use the bar method.</p>
<p>Percentages In Year 6 we solve problems involving the calculation of percentages and the use of percentages for comparison.</p>	<p>We still need to use visual images to support our calculations. The bar model, paper strips or number line should be used to estimate our answer first.</p> <p>A good question to ask:</p> <p>If I knock 10% off today, and put 10% back on tomorrow what happens?</p>